International Journal of Theoretical Physics, Vol. 17, No. 1 (1978), pp. 21–32

Nuclear and Atomic Models

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Received May 10, 1977

A theorem concerning fermion interaction is postulated and applied to the problems of atomic (electronic) and nuclear physics. Model building based *solely* upon the postulate that *adjacent like fermions must be singlet paired* accounts for the closed shells of *both* nuclear and atomic structure. The implied antiferromagnetic FCC lattice of protons and neutrons in alternating layers has been found previously to be the lowest-energy solid configuration of nuclear matter ($N = P$) (Canuto and Chitre, 1974). The buildup of the FCC lattice from a central tetrahedron reproduces all of the shells and subshells of the isotropic harmonic oscillator, which of course is the basis for the shell model. In atomic structure, the singlet pairing of adjacent electrons implies closed-shell structures uniquely at the six noble gases and the three noble metals, Ni, Pd, and Pt. The basis for the postulate concerning fermions is found in terms of classical electrodynamics; it is a microscopic corollary of Biot-Savart's law that parallel currents attract whereas antiparallel currents repel.

1. INTRODUCTION

Model building has long been a useful approach in a number of areas of physical science. Particularly in relation to macromolecule and crystal structures, scale models have been necessary as first approximations before precise experimental data become available. Even the problems of the structure of the smaller organic and inorganic molecules can be usefully approached through "naive," three-dimensional models—often with results that closely mimic subsequent experimental measurements (e.g., Bent, 1963; 1970) and that rival much more sophisticated mathematical theories in reproducing empirical values (Gillespie, 1972). Less successful to date, yet of the same general nature, are the alpha-particle and cluster models of nuclear physics (e.g., Goldhammer, 1963; Hauge et al., 1971)—where again "naive" macro-

0020-7748/78/0100-0021$05.00/0

scopic models serve as the basis from which more precise calculations can proceed. As Pauling has emphasized with regard to his own "helion" cluster theory, model building is not intended as a substitute for quantum-mechanical statistical work (Pauling and Robinson, 1975), but rather is intended as a first-order approximation and as a concrete conceptual framework from which mathematical implications can be abstracted. The following observations on nuclear and atomic models are presented in that spirit.

2. NUCLEAR STRUCTURE

The principal difference between the nuclear system of fermions (nucleons) and the atomic system of fermions (electrons) is the difference between a system regulated predominantly by a short-range interparticle force (the nuclear force) versus a system regulated by a long-range central force (the electrostatic force field of the nuclear charge) (Weisskopf, 1968). On the one hand, there is a homogeneity throughout the nucleus resulting in a roughly even distribution of charge in the nuclear interior, whereas in the atom there is a much higher concentration of electron mass and charge near the nucleus than at the outer edges of the electron configuration. Furthermore, the nuclear and atomic systems differ in that the hard-core nucleon radius (0.8 fm) is a sizable fraction of the nuclear radius (< 6.0 fm); whereas the electron, if it has a hard-core radius at all (e.g., Yadava, 1976), has a radius that is five orders of magnitude smaller than the atom itself (1 fm:1 Å). As shown in Table I, using accepted values for nucleon and nuclear dimensions, nuclei are

TABLE 1. Calculation of the volume occupied by nucleons in representative nuclei as compared to the volume of the nuclei themselves shows that nuclei are extremely dense. Nuclear radial values from Hofstadter (1957).

Nucleus	Radius (fm)	Volume of sphere (fm^3)	Volume of N nucleons (fm^3)	Volume needed for close packing N nucleons (c/.74)	Percent of nuclear volume filled by N close packed nucleons
H^1	0.80	2.15	2.15	—	—
He^4	1.61	17.48	8.60	11.62	66
O^{16}	2.64	77.08	34.40	44.86	58
Si^{28}	3.04	117.69	60.20	81.35	69
Ca^{40}	3.52	182.68	86.00	116.22	64
Ni^{58}	3.93	254.26	124.70	168.51	66
Sr^{88}	4.14	297.24	189.20	242.16	64
In^{115}	4.50	381.70	247.25	334.12	88
Au^{197}	5.32	630.70	423.55	572.30	91
Pb^{208}	5.42	666.97	447.20	604.32	91

between 50% and 90% filled with nucleon matter. These calculations should not be taken too seriously since neither the nature of the nuclear surface nor a sophisticated description of nucleon structure is taken into account, but the implications are nonetheless clear: Nuclei are fairly dense entities. Experimentally, it is known that low-energy protons injected into nuclei (at 20–30 MeV, which is equivalent to the energy of nucleons within the nucleus) collide and disperse their momentum within one fermi of the nuclear surface (discussed in detail by Weisskopf, 1972)—indicating that the nucleus is at least a dense liquid. On the other hand, the shell, or independent-particle, model has had tremendous success in explaining nuclear properties—notably, angular momentum values, magnetic moments, and parities—by assuming that the nucleus is a diffuse *gas*. The obvious question is whether or not the empirical data on nuclear density, etc. and the shell theory are in fact contradictory. Bohr and Mottelson (1969) have simply declared that the nucleons have long mean free paths within nuclei (as is necessary for their "unified" theory), relevant low-energy experimental data to the contrary notwithstanding. Using unorthodox nucleon and nuclear dimensions, Jammer (1966) has calculated that the nucleons occupy only 1/50th of the volume of a nucleus. Weisskopf (1951) has speculated that the exclusion principle "prohibits" collisions between nucleons in a dense nuclear interior—thereby allowing for a relatively long mean free path, i.e., gaslike behavior within a liquid medium. Others have assumed that the nucleon's sometimes wavelike nature circumvents this problem entirely. Yet, each of these "solutions" is open to serious criticism; there remains the very real *physical* problem of how relatively large nucleons "orbit" within a relatively small nuclear volume. It is somewhat paradoxical, but the apparent contradictions between liquid and gaseous nuclear models are resolvable in terms of a crystalline *solid* which reproduces geometrical closed shells at the shell model "magic" numbers. As shown below, the generation of geometrical shells in a particular lattice structure circumvents the basic theoretical need for a *gaseous* shell model.

Although most theoretical work concerning the solidification of nuclear matter shows normal nuclei to be in a liquid phase (Canuto, 1974; 1975), there are so many uncertainties in such theoretical calculations that the results vary by more than an order of magnitude. Regardless of the problematical numerical value for solidification—which may indeed be as low as the density of normal nuclear matter (Calogero et al., 1975)—the lowest-energy crystalline packing scheme when solidification densities are attained is likely to be an antiferromagnetic FCC structure with alternating layers of protons and neutrons (Canuto and Chitre, 1973; 1974). Using that crystal configuration as a guide and building from a central tetrahedron (i.e., the doubly magic closed shell ${}^{2}_{2}He^{4}$ nucleus), model building produces some interesting correlations with the shell model and empirical data (Cook, 1976).

As shown in Figure 1, symmetrical build-up from the helium tetrahedron gives closed-shell structures (with equivalent x, y, and z dimensions) at 4, 16, 40, 80, 140, and 224 particles, corresponding to the doubly magic shell numbers predicted by an isotropic harmonic oscillator—that is, closed shells at 2, 8, 20, 40, 70, and 112 protons or neutrons. As is well known from work on the shell model, appropriate splitting of the harmonic oscillator shells into subshells allows for a rationalization of the empirically known magic numbers (Mayer and Jensen, 1955). A similar splitting of the FCC lattice shells produces the same results. As shown in Figure 2, a nucleon's angular momentum quantum value j in this model is dependent upon the nucleon's distance from

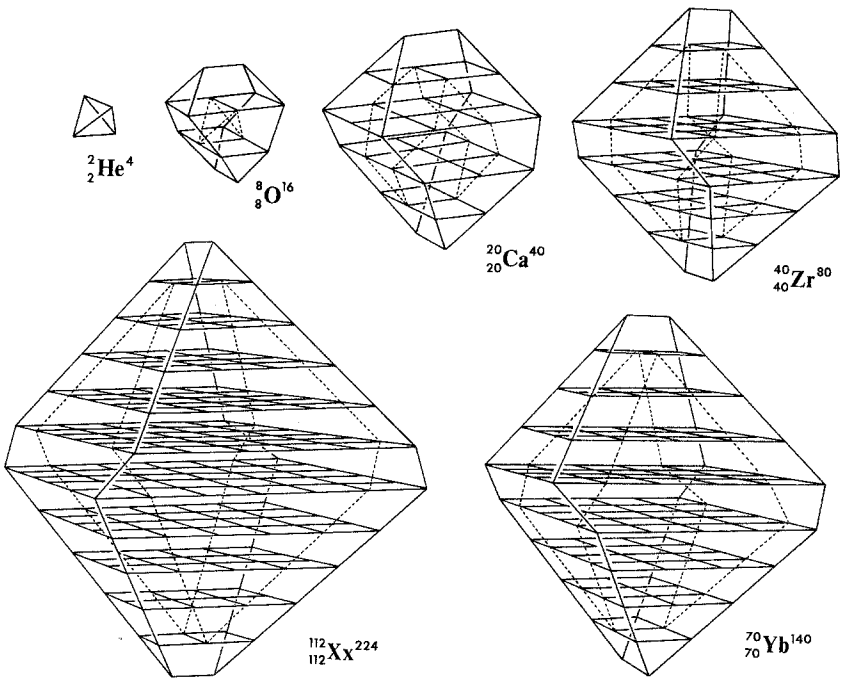

Fig. 1. The buildup of the antiferromagnetic lattice from a central tetrahedron. All of the figures have $x = y = z$ dimensions and tetrahedral symmetry. The three larger ones are the "doubly magic" nuclei predicted by an isotropic harmonic oscillator. Since the larger nuclei need excess neutrons, the harmonic oscillator $N = P$ structures are not found in nature; "magic" nuclei arise by completing the largest horizontal "layer" of nucleons in each figure [as shown for $^{70}_{70}Yb^{140}$—giving a (70 + 12 =) 82 "magic" neutron shell]. These two-dimensional drawings unfortunately obscure the simplicity and perfect $x = y = z$ symmetry of the figures, which can be appreciated only in three-dimensional models. Nonetheless, the shell and subshell buildup is evident. These structures are the inevitable result of building nuclei according to the postulate that adjacent like fermions be singlet paired.

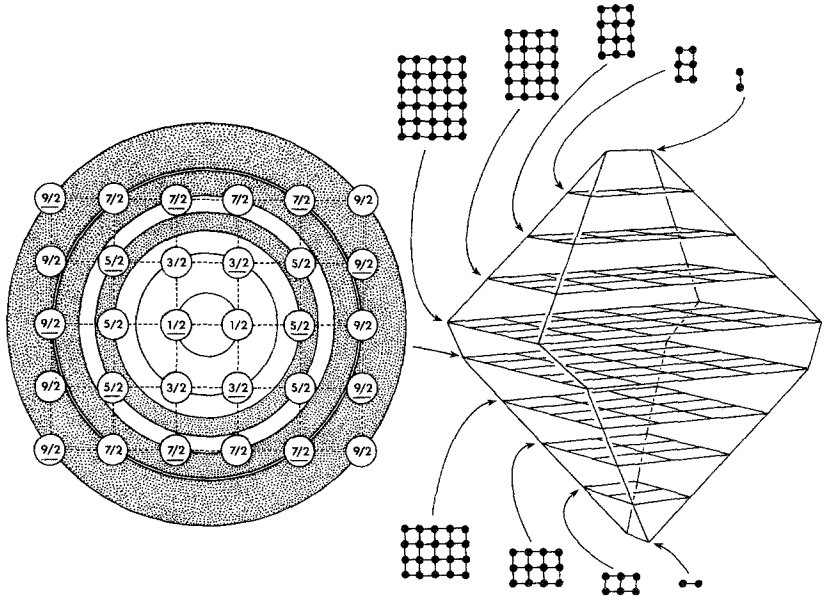

Fig. 2. The $^{70}_{70}Yb^{140}$ nucleide. The horizontal layers alternate between protons and neutrons. One of the central layers is enlarged on the left to show the dependency of the nucleon j value on its distance from the central vertical axis. The spin of adjacent nucleons in each level is always opposite in sign—spin up being underlined and spin down not underlined (or vice versa).

the vertical axis—in analogy with classical mechanics. The correct number of nucleons of whatever j value in each shell (quantum value n as shown in Figure 1) is then found in any given nucleus (Cook, 1975). This nucleon labeling process naturally gives rise to a series of "subshells" (according to the angular momentum value of the nucleons) which break down into the *same* series as the harmonic oscillator: 2, 6, 8, 14, 18, 20, 28, 34, 38, 40, 50, 58, 64, 70, 82, 92, 100, 106, 110, 112, 126, ... Although the FCC structure does not uniquely predict the six (or seven) known magic numbers, it does predict the first three (the only existent $N = P$ doubly magic shells, $^{2}_{2}He^{4}$, $^{8}_{8}O^{16}$, and $^{20}_{20}Ca^{40}$) and all of the subshells of the isotropic harmonic oscillator —among which three more (50, 82, and 126) are unusually stable. Furthermore, it is to be noted that of the full array of 22 harmonic oscillator subshells (2, . . . , 126) fully 14 show "magic" stability of one kind or another depending upon which criterion of "magicness" we select (number of known isotopes or isotones, number of stable isotopes or isotones, quadrupole moment values, neutron separation energies, excitation energy of the first 2+ state, etc.) (Cook, 1976). Although the exceptional stability of 50, 82, and 126 is not

easily accounted for in the FCC model, the appearance of a large number of relatively stable subshells is comprehensible within the framework of the FCC lattice buildup procedure: *The closing of a subshell, as in the completion of a face of any crystal structure, maximizes the two-body bonding of the nuclear particles.* Indeed, the significance of the conventional shell model lies not primarily in its identification of "magic" shells—even the number of which is debatable—but in its use of the spin-orbit coupling model, which is an integral part of the FCC model as well. This means that the angular momentum, magnetic moment, and parity predictions—achieved through the use of the spin-orbit coupling model in conventional shell theory—are also fundamentally derivable through the FCC lattice model's use of a similar spin-orbit coupling model.

In summary, what the FCC lattice allows for is a nuclear model that has a nucleon labeling system and a nuclear subshell system *identical* with the shell model, but that also retains some of the features of the seemingly contradictory "collective" or liquid drop model (constant nuclear density, nearest neighbor nucleon interaction, and saturation of the nuclear force) and the cluster models (an alpha clustering implicit to the FCC crystal structure) (Lezuo, 1974).

The antiferromagnetic FCC lattice with alternating layers of protons and neutrons at first appears to be an inexplicable, somewhat bizarre crystal structure for nuclear matter, but inspection of the crystal shows that its low-energy nature is due to the favorable magnetic interaction produced between adjacent nucleons in each level (see Figure 2). The simplicity of the crystal structure becomes evident when the constraints that make it possible are made explicit: the antiferromagnetic FCC lattice with alternating layers of protons and neutrons is implied in a close-packed configuration of nucleons by stipulating that *adjacent like fermions must be singlet paired.* The raison d'être for the postulate is presumably due to the magnetic interaction of the nucleons. Alternatively, the postulate can be viewed as a generalization of the version of the exclusion principle that states that like fermions "adjacent" within an energy level must be singlet paired. Either way, this theorem is necessarily applicable to electrons and the problems of atomic (and molecular) structure.

3. ATOMIC STRUCTURE

Before the emergence of quantum theory, Lewis (1916) and Langmuir (1919) developed a theory of atomic structure (through argon) based upon the geometrical arrangement of electrons around the nucleus—the so-called "static atom" model. Their depiction of the electron as a static point charge became untenable with knowledge of the electron's intrinsic and orbital

magnetic moments and angular momenta, and the theory as a whole has been discarded. Nonetheless, adopting a more modern conception of the electron as a small charge moving within a larger "probability cloud," their prequantum geometrical insights still have some value.

As mentioned above, the atomic system is guided by the spherically symmetrical electrostatic field of the nucleus. Consequently, the adjacency of nearest neighbors will be adjacency in a spherical coordinate system—spin up and spin down always in relation to the nuclear core. The first two electrons of the helium inert closed shell will be opposite spin electrons when side by side, yet, when placed on opposite sides of the helium nucleus to reduce the electrostatic repulsion between them, they *appear* to be like-spinning electrons. As shown in Figure 3, the helium electron spheres are assumed to take on roughly hemispherical shapes due to the electrostatic attraction of the nucleus. All electrons that follow are given their own individual spherical volume; only the helium electrons, which straddle the nuclear charge, are hemispherical.

Subsequent electron buildup onto the helium core is straightforward. The size of the electron "cloud" is allowed to vary—the inner core electrons occupying considerably less volume than the outer shell electrons owing to their proximity to the nuclear charge—but the electrons of any given shell (any given distance to the nucleus) are seen as identical in size. By demanding that nearest-neighbor electrons be singlet paired, definite geometrical closed shells arise at certain numbers of electron spheres—namely, at the known inert gases. Neon is constructed with the cubic arrangement of eight equivalent spheres around the spherical helium core. Argon arises with the cubic arrangement of eight more spheres around the two inner core structures of 2 and 8 electrons—now reduced in size owing to the much stronger +18 nuclear charge (see Figure 4). Alternative and/or intermediate structures such

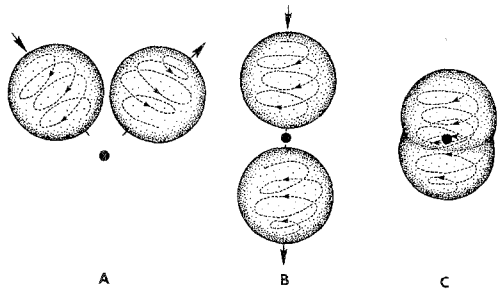

Fig. 3. (A) The singlet paired electrons of helium (the depiction of the electron after Bunge, 1955). (B) The same electrons on opposite sides of the nucleus. (C) The likely final configuration where the helium electron clouds each occupy roughly a hemispherical volume.

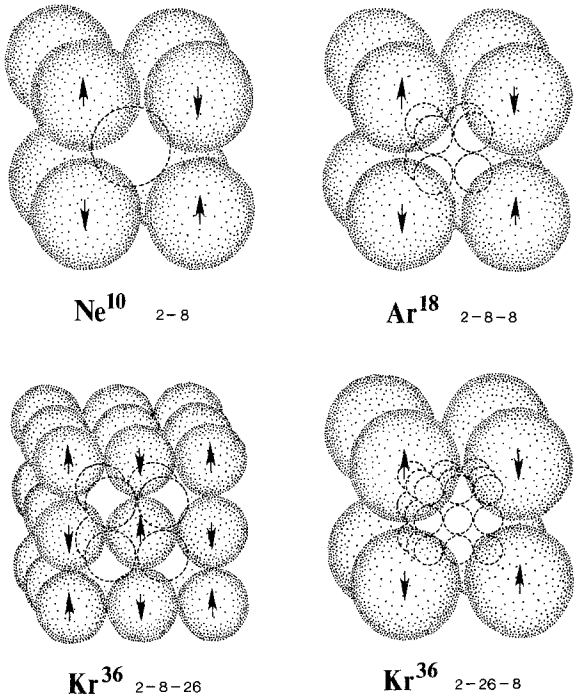

Fig. 4. The neon, argon, and krypton closed shells. The neon and argon models are similar to those of the "static atom" theory of Lewis (1916) and Langmuir (1919). Krypton can be arranged as three shells of 2, 8, and 26 electrons or 2, 26, and 8 electrons—a straightforward geometrical argument apparently overlooked by Lewis and Langmuir. All electrons in these models have opposite spin nearest neighbors.

as octahedrons are prohibited by the stipulation that nearest neighbors be singlet paired. Krypton can be made from an arrangement of inner cores of 2 and 8 electrons with an external shell of 26 electrons in a cubic configuration—all with opposite spin nearest neighbors. The krypton closed shell can be rearranged to an equally plausible 2, 26, 8 structure to give an eight-electron valence shell (Figure 4). Xenon can be built from a 2, 26, 26 sphere arrangement—again with inner shells reduced to appropriate size with all electrons with opposite spin nearest neighbors. To produce an eight-electron valence shell, shells of 2, 26, 18, and 8 electrons can be constructed (Figure 5). Finally, radon can be constructed as a 2, 8, 26, 50 electron structure or, alternatively, 2, 26, 50, 8.

The cubic shape of all of these inert gases (except helium) is significant in that it allows for a prediction concerning their crystal structures: face-centered-cubic close packing is implied. Hexagonal close packing, which is

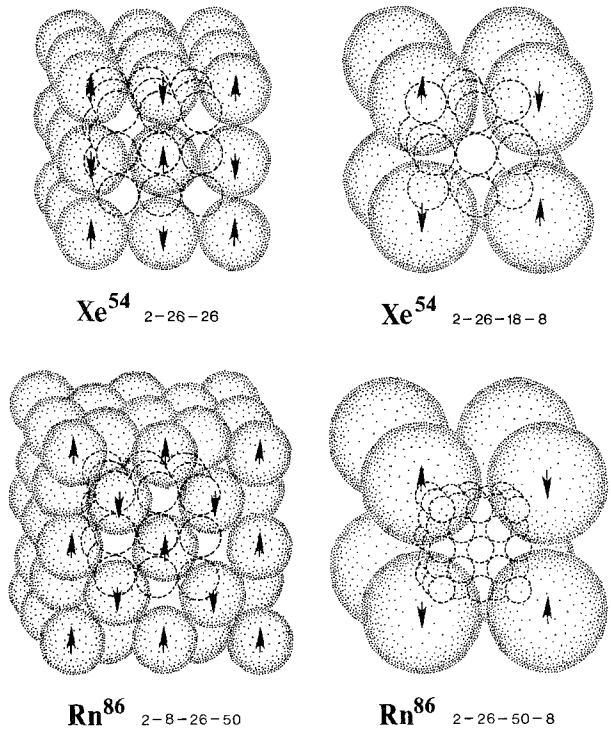

Fig. 5. The xenon (2, 26, 26 or 2, 26, 18, 8 electron shells) and radon (2, 8, 26, 50 or 2, 26, 50, 8 electron shells) inert gas structures.

deduced from quantum theoretical models (e.g., Cuthbert and Linnett, 1958), is impossible for cubic structures. Empirically, all of the inert gases except helium are found to crytallize in the FCC configuration (Smith, 1971).

An important remaining question is whether or not these closed-shell structures are unique or merely six of a large number of possibilities derived from the "expanded" exclusion principle. With one qualification, the answer is that they are unique: These cubic structures are the *only* symmetrical ($x = y = z$), closed-shell structures with all neighboring electrons singlet paired—provided that a rearrangement with an outer shell of eight electrons is possible. Without this qualification, three other similar closed shells (with more than eight electrons in the most external shell) arise at the noble metals, Ni^{28}, Pd^{46}, and Pt^{78}—*all three of which also crystallize in the FCC array* (see Figure 6).

The above geometrical description of atomic structure is still a long way from solving the long-standing problem of a self-consistent explanation of

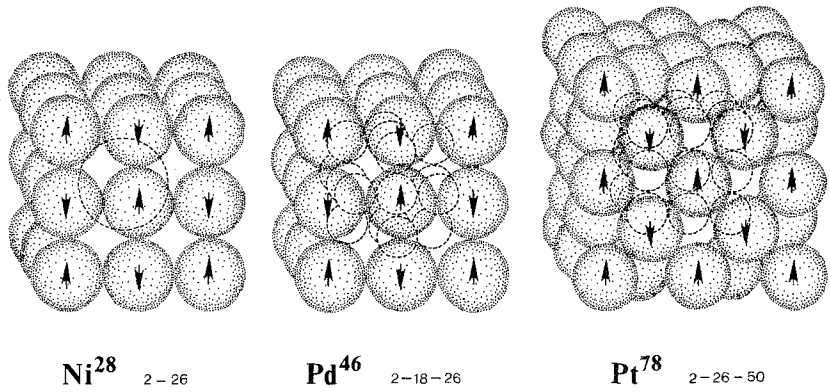

Fig. 6. Stipulating only that electrons must be singlet paired with nearest neighbors, three structures other than the noble gases are found: the noble metals, nickel, palladium, and platinum. These three (and the copper–silver–gold group with one excess "conductance" electron) crystallize in the FCC array, as do the noble gases.

all atomic crystal patterns, but it does provide a rationalization of the crystal structure of the noble gases (a particularly vexing problem in quantum chemistry since the noble gases offer a theoretically simple case, yet all approaches lead to HCP predictions) (Smith, 1971) and the noble metals and, more importantly, offers a geometric rationalization of the basic periodicity of the periodic chart of elements. Together with the fundamentally similar electron-repulsion theory of molecular structure by Gillespie (1972), these models may provide an alternative conceptual framework to the wave mechanical depictions of conventional atomic and molecular theory.

4. CONCLUSION

The theoretical strength of these nuclear and atomic models lies in the fact that a *single* principle of particle interaction—the singlet pairing of adjacent like fermions—provides a rationalization of *both* the nuclear and the atomic closed shells. Because of the intrinsic differences between an all-electron, centrally organized atomic structure versus a proton-and-neutron, homogenous nuclear structure, the atomic and nuclear systems take on very different configurations. Nonetheless, being *fermion* systems, they are both guided by fundamentally similar dynamics.

By implication, conventional theory is challenged on a number of grounds by this theory, but the geometrical models need not be viewed as fundamentally contrary to known quantum mechanical laws. Certainly the *individual* electron is adequately described through quantum mechanics. Yet, contrary

to conventional theory and the usual extrapolation from known one-electron features to purely theoretical many-electron dynamics, the many-electron system—which quantum theory is incapable of dealing with without empirically guided approximation methods—may be controlled primarily by electromagnetic effects, i.e., by the electrostatic repulsion between electron clouds (Gillespie, 1972) and the slight magnetic attraction between singlet paired electron clouds (Heitzmann, 1975). Such electromagnetic properties of the many-electron system are explicitly not a part of the first-order quantum-mechanical description. The possibility that quantum mechanics offers a valuable description of the lone electron (and electron transition processes), but fails fundamentally to account for atomic structure (the many-electron system) should not be overlooked in light of the *ad hoc* exclusion principle, *ad hoc* Hund's rules, *semiempirical* approximation methods, and less than rigorously self-consistent theories of conventional atomic and molecular physics.

In terms of nuclear theory, the FCC lattice model is actually based upon the central premise of the shell model, viz., the spin-orbit coupling model and its implied system of nucleon subshells. Consequently, it is less unconventional than the atomic models despite the unconventional implication of a solid phase state. It should be noted here that the apparently liquid and even gaseous characteristics of nuclei indicated by virtually all *high*-energy experimental work may be a result of the nature of the *nucleons themselves* and not due to the nature of *nuclear structure*. The "cloudy crystal ball" and gaseous effects may be the result of the inherently liquid or gaseous nature of the individual nucleons (confined to a small volume, $r = 0.8$ fm) rather than the result of a liquid or gaseous movement of the nucleons throughout the *nuclear* volume.

Previously, geometrical models have been proposed to account for atomic periodicity (Stevens, 1966; Luder, 1967; Lewis, 1916; Langmuir, 1919) and for nuclear periodicity (Pauling, 1965; Anagnostatos, 1973; Lezuo, 1974 and 1975; Fuller, 1975). Each, however, has been a model uniquely applicable to either the atomic or the nuclear system. The models presented here, on the other hand, rationalize the periodicity of both systems in a unified approach. Furthermore, the underlying theorem concerning fermion interaction is comprehensible in terms of classical electrodynamics: Biot-Savart's law that parallel (like) currents attract whereas antiparallel (unlike) currents repel would be a macroscopic manifestation of the same principle found between neighboring like fermions.

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